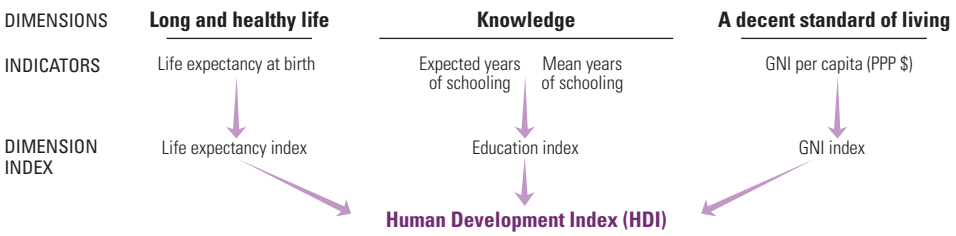
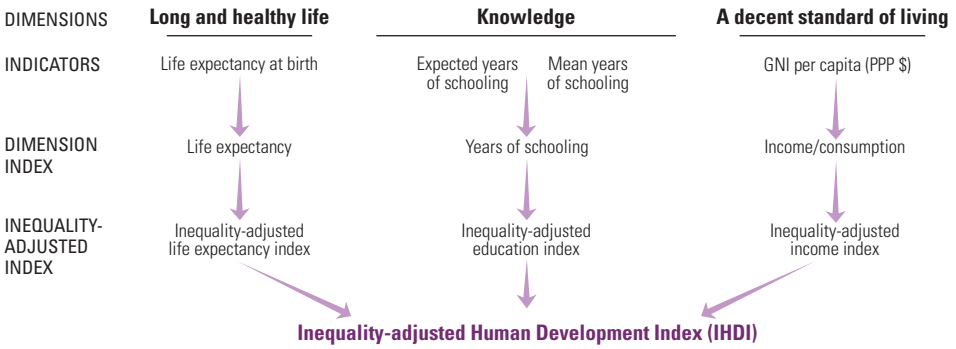
Technical notes

Calculating the human development indices—graphical presentation

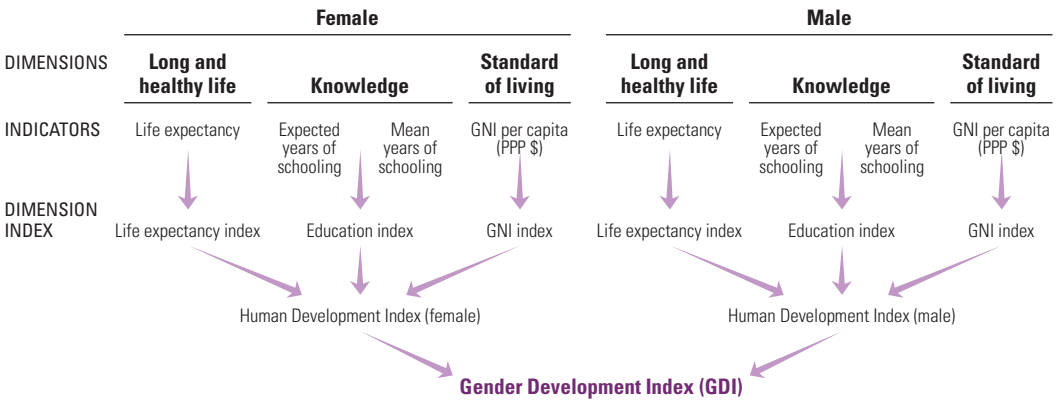
Human Development Index (HDI)



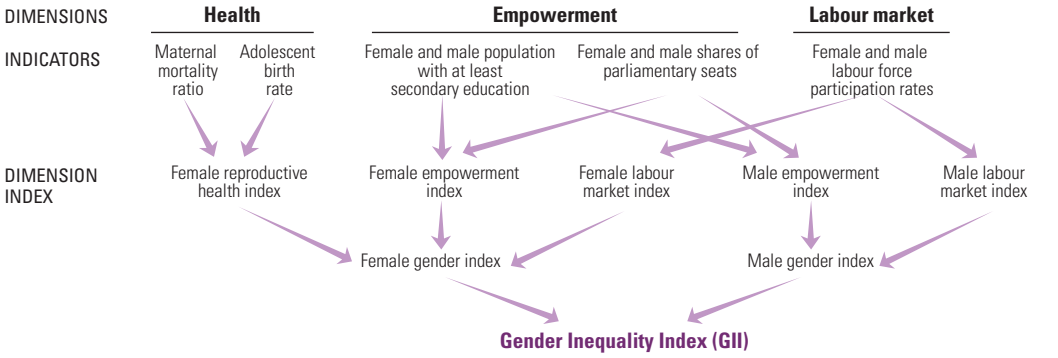
Inequality-adjusted Human Development Index (IHDI)



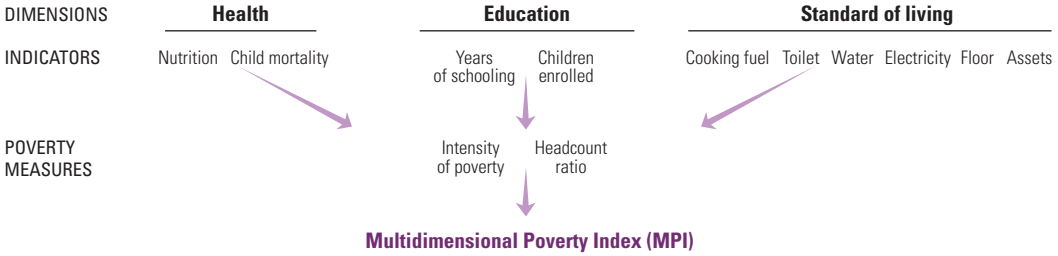
Gender Development Index (GDI)



Gender Inequality Index (GII)



Multidimensional Poverty Index (MPI)



Technical note 1. Human Development Index

The Human Development Index (HDI) is a summary measure of achievements in three key dimensions of human development: a long and healthy life, access to knowledge and a decent standard of living. The HDI is the geometric mean of normalized indices for each of the three dimensions.

Data sources

- Life expectancy at birth: UNDESA (2017).
- Expected years of schooling: UNESCO Institute for Statistics (2018), ICF Macro Demographic and Health Surveys, United Nations Children's Fund (UNICEF) Multiple Indicator Cluster Surveys and OECD (2017).
- Mean years of schooling: UNESCO Institute for Statistics (2018), Barro and Lee (2016), ICF Macro Demographic and Health Surveys, UNICEF Multiple Indicator Cluster Surveys and OECD (2017).
- GNI per capita: World Bank (2018), IMF (2018) and United Nations Statistics Division (2018).

Steps to calculate the Human Development Index

There are two steps to calculating the HDI.

Step 1. Creating the dimension indices

Minimum and maximum values (goalposts) are set in order to transform the indicators expressed in different units into indices between 0 and 1. These goalposts act as the "natural zeros" and "aspirational targets," respectively, from which component indicators are standardized (see equation 1 below). They are set at the following values:

Dimension	Indicator	Minimum	Maximum
Health	Life expectancy (years)	20	85
Education	Expected years of schooling (years)	0	18
	Mean years of schooling (years)	0	15
Standard of living	Gross national income per capita (2011 PPP $)	100	75,000

The justification for placing the natural zero for life expectancy at 20 years is based on historical evidence that no country in the 20th century had a life expectancy of less than 20 years (Maddison, 2010; Oeppen and Vaupel, 2002; Riley, 2005). Maximum life expectancy is set at 85, a realistic aspirational target for many countries over the last 30 years. Due to constantly improving living conditions and medical advances, life expectancy has already come very close to 85 years in several economies: Hong Kong, China (Special Administrative Region) (84.1 years) and Japan (83.9 years).

Societies can subsist without formal education, justifying the education minimum of 0 years. The maximum for expected years of schooling, 18, is equivalent to achieving a master's degree in most countries. The maximum for mean years of schooling, 15, is the projected maximum of this indicator for 2025.

The low minimum value for gross national income (GNI) per capita, $100, is justified by the considerable amount of unmeasured subsistence and nonmarket production in economies close to the minimum, which is not captured in the official data. The maximum is set at $75,000 per capita. Kahneman and Deaton (2010) have shown that there is virtually no gain in human development and well-being from annual income per capita above $75,000. Currently, only four countries (Brunei Darussalam, Liechtenstein, Qatar and Singapore) exceed the $75,000 income per capita ceiling.

Having defined the minimum and maximum values, the dimension indices are calculated as:

$$\text{Dimension index} = \frac{\text{actual value} - \text{minimum value}}{\text{maximum value} - \text{minimum value}}. \qquad (1)$$

For the education dimension, equation 1 is first applied to each of the two indicators, and then the arithmetic mean of the two resulting indices is taken. Using the arithmetic mean allows perfect substitutability between mean years of schooling and expected years of schooling. Many developing countries have low school attainment among adults but are eager to achieve universal primary and secondary school enrolment.

Because each dimension index is a proxy for capabilities in the corresponding dimension, the transformation function from income to capabilities is likely to be concave (Anand and Sen 2000)—that is, each additional dollar of income has a smaller effect on expanding capabilities. Thus for income, the natural logarithm of the actual, minimum and maximum values is used.

Step 2. Aggregating the dimensional indices to produce the Human Development Index

The HDI is the geometric mean of the three dimensional indices:

$$HDI = (I_{Health} \cdot I_{Education} \cdot I_{Income})^{1/3}$$

Example: Egypt

Indicator	Value
Life expectancy at birth (years)	71.7
Expected years of schooling (years)	13.1
Mean years of schooling (years)	7.2
Gross national income per capita (2011 PPP $)	10,355

Note: Values are rounded.

$$\text{Health index} = \frac{71.661 - 20}{85 - 20} = 0.7948$$

$$\text{Expected years of schooling index} = \frac{13.0898 - 0}{18 - 0} = 0.7272$$

$$\text{Mean years of schooling index} = \frac{7.218 - 0}{15 - 0} = 0.4812$$

$$\text{Education index} = \frac{0.4812 + 0.7272}{2} = 0.6042$$

$$\text{Income index} = \frac{\ln(10{,}355) - \ln(100)}{\ln(75{,}000) - \ln(100)} = 0.7009$$

$$\text{Human Development Index} = (0.7948 \cdot 0.6042 \cdot 0.7009)^{1/3} = 0.696$$

Methodology used to express income

The World Bank's 2018 World Development Indicators database contains estimates of GNI per capita in constant 2011 purchasing power parity (PPP) terms for many countries. For countries missing this indicator (entirely or partly), the Human Development Report Office (HDRO) calculates it by converting GNI per capita from current to constant terms using two steps. First, the value of GNI per capita in current terms is converted into PPP terms for the base year (2011). Second, a time series of GNI per capita in constant 2011 PPP terms is constructed by applying the real growth rates to the GNI per capita in PPP terms for the base year. The real growth rate is implied by the ratio of the nominal growth of GNI per capita in current local currency terms to the GDP deflator.

For several countries without a value of GNI per capita in constant 2011 PPP terms for 2017 reported by the World Bank, the International Monetary Fund (IMF)–projected real growth rates of GDP are applied to the most recent GNI values in constant PPP terms. The IMF-projected growth rates are calculated based on local currency terms and constant prices rather than in PPP terms. This avoids mixing the effects of the PPP conversion with those of real growth of the economy.

Official PPP conversion rates are produced by the International Comparison Program, whose surveys periodically collect thousands of prices of matched goods and services in many countries. The last round of this exercise refers to 2011 and covered 199 countries.

Estimating missing values

For a small number of countries missing one of the four indicators, the HDRO estimated the missing values using cross-country regression models.

In this Update expected years of schooling were estimated for Bahamas, Dominica, Equatorial Guinea, Haiti, Libya, Papua New Guinea, Tonga, Trinidad and Tobago, and Vanuatu. Mean years of schooling were estimated for Eritrea, Grenada and Saint Kitts and Nevis.

Country groupings

The 2014 Human Development Report introduced fixed cutoff points for four categories of human development achievements. The cutoff points (COP) are the HDI values calculated using the quartiles (q) from the distributions of the component indicators (life expectancy at birth [LE], expected years of schooling [EYS], mean years of schooling [MYS] and GNI per capita [$GNIpc$]):

$$COP_q = HDI\,(LE_q, EYS_q, MYS_q, GNIpc_q),\ q = 1,2,3.$$

For example, LE1, LE2 and LE3 denote three quartiles of the distribution of life expectancy across countries. The resulting HDI values are averaged over 2004–2013.

This Update keeps the same cutoff points of the HDI for grouping countries that were introduced in the 2014 Report:

Very high human development	0.800 and above
High human development	0.700–0.799
Medium human development	0.550–0.699
Low human development	Below 0.550

Technical note 2. Inequality-adjusted Human Development Index

The Inequality-adjusted Human Development Index (IHDI) adjusts the Human Development Index (HDI) for inequality in the distribution of each dimension across the population. It is based on a distribution-sensitive class of composite indices proposed by Foster, Lopez-Calva and Szekely (2005), which draws on the Atkinson (1970) family of inequality measures. It is computed as a geometric mean of inequality-adjusted dimensional indices.

The IHDI accounts for inequalities in HDI dimensions by "discounting" each dimension's average value according to its level of inequality. The IHDI equals the HDI when there is no inequality across people but falls below the HDI as inequality rises. In this sense, the IHDI measures the level of human development when inequality is accounted for.

Data sources

Since the HDI relies on country-level aggregates such as national accounts for income, the IHDI must draw on additional sources of data to obtain insights into the distribution. The distributions are observed over different units—life expectancy is distributed across a hypothetical cohort, while years of schooling and income are distributed across individuals.

Inequality in the distribution of HDI dimensions is estimated for:

- Life expectancy, using data from abridged life tables provided by UNDESA (2017). This distribution is presented over age intervals (0–1, 1–5, 5–10, ... , 85+), with the mortality rates and average age at death specified for each interval.
- Mean years of schooling, using household surveys data harmonized in international databases, including the Luxembourg Income Study, Eurostat's European Union Survey of Income and Living Conditions, the World Bank's International Income Distribution Database, ICF Macro Demographic and Health Surveys, United Nations Children's Fund Multiple Indicators Cluster Survey, the Center for Distributive, Labor and Social Studies and the World Bank's Socio-Economic Database for Latin America and the Caribbean and the United Nations University's World Income Inequality Database.
- Disposable household income or consumption per capita using the above listed databases and household surveys—and for a few countries, income imputed based on an asset index matching methodology using household survey asset indices (Harttgen and Vollmer 2013).

A full account of data sources used for estimating inequality in 2017 is available at http://hdr.undp.org/en/statistics/ihdi/.

Steps to calculate the Inequality-adjusted Human Development Index

There are three steps to calculating the IHDI.

Step 1. Estimating inequality in the dimensions of the Human Development Index

The IHDI draws on the Atkinson (1970) family of inequality measures and sets the aversion parameter ε equal to 1.[1] In this case the inequality measure is $A = 1 - g/\mu$, where g is the geometric mean and μ is the arithmetic mean of the distribution. This can be written as:

$$A_x = 1 - \frac{\sqrt[n]{X_1 \dots X_n}}{\bar{X}} \qquad (1)$$

where $\{X_1, \dots, X_n\}$ denotes the underlying distribution in the dimension of interest. A_x is obtained for each variable (life expectancy, mean years of schooling and disposable household income or consumption per capita).

The geometric mean in equation 1 does not allow zero values. For mean years of schooling one year is added to all valid observations to compute the inequality. Income per capita outliers—extremely high incomes as well as negative and zero incomes—were dealt with by truncating the top 0.5 percentile of the distribution to reduce the influence of extremely high incomes and by replacing the negative and zero incomes with the minimum value of the bottom 0.5 percentile of the distribution of positive incomes. Sensitivity analysis of the IHDI is given in Kovacevic (2010).

Step 2. Adjusting the dimension indices for inequality

The inequality-adjusted dimension indices are obtained from the HDI dimension indices, I_x, by multiplying them by $(1 - A_x)$, where A_x, defined by equation 1, is the corresponding Atkinson measure:

$$I_x^* = (1 - A_x) \cdot I_x .$$

The inequality-adjusted income index, I^*_{income}, is based on the index of logged income values, I_{income^*} and inequality in income distribution computed using income in levels. This enables the IHDI to account for the full effect of income inequality.

Step 3. Combining the dimension indices to calculate the Inequality-adjusted Human Development Index

The IHDI is the geometric mean of the three dimension indices adjusted for inequality:

$$IHDI = (I^*_{Health} \cdot I^*_{Education} \cdot I^*_{Income})^{1/3} =$$

$$[(1 - A_{Health}) \cdot (1 - A_{Education}) \cdot (1 - A_{Income})]^{1/3} \cdot HDI.$$

The loss in the Human Development Index value due to inequality is:

$$Loss = 1 - [(1 - A_{Health}) \cdot (1 - A_{Education}) \cdot (1 - A_{Income})]^{1/3}.$$

Coefficient of human inequality

An unweighted average of inequalities in health, education and income is denoted as the coefficient of human inequality. It averages these inequalities using the arithmetic mean:

$$Coefficient\ of\ human\ inequality = \frac{A_{Health} + A_{Education} + A_{Income}}{3}.$$

When all inequalities in dimensions are of a similar magnitude the coefficient of human inequality and the loss in HDI value differ negligibly. When inequalities differ in magnitude, the loss in HDI value tends to be higher than the coefficient of human inequality.

Notes on methodology and caveats

The IHDI is based on the Atkinson index, which satisfies subgroup consistency. This property ensures that improvements (deteriorations) in the distribution of human development within only a certain group of the society imply improvements (deteriorations) in the distribution across the entire society.

The main disadvantage is that the IHDI is not association sensitive, so it does not capture overlapping inequalities. To make the measure association sensitive, all the data for each individual must be available from a single survey source, which is not currently possible for a large number of countries.

Example: Madagascar

Indicator	Value	Dimension index	Inequality measure[a] (*A*)	Inequality-adjusted index (I^*)
Life expectancy (years)	66.3	0.7125	0.213	(1–0.213) · 0.7125 = 0.5607
Expected years of schooling (years)	10.6	0.5872	—	—
Mean years of schooling (years)	6.1	0.4097	0.350	—
Education index	—	0.4985	0.350	(1–0.350) · 0.4985 = 0.3240
Gross national income per capita (2011 PPP $)	1,358	0.3940	0.204	(1–0.204) · 0.394 = 0.3136

Human Development Index	Inequality-adjusted Human Development Index
$(0.7125 \cdot 0.4986 \cdot 0.3940)^{1/3} = 0.5191$	$(0.5607 \cdot 0.3240 \cdot 0.3136)^{1/3} = 0.3848$
Loss due to inequality (%)	**Coefficient of human inequality (%)**
$100 \cdot \left(1 - \frac{0.385}{0.519}\right) = 25.9$	$\frac{100 \cdot (0.213 + 0.350 + 0.204)}{3} = 25.6$

Note: Values are rounded.
a. Inequalities are estimated from micro data.

Technical note 3. Gender Development Index

The Gender Development Index (GDI) measures gender inequalities in achievement in three basic dimensions of human development: health, measured by female and male life expectancy at birth; education, measured by female and male expected years of schooling for children and female and male mean years of schooling for adults ages 25 years and older; and command over economic resources, measured by female and male estimated earned income.

Data sources

- Life expectancy at birth: UNDESA (2017).
- Expected years of schooling: UNESCO Institute for Statistics (2018), ICF Macro Demographic and Health Surveys, United Nations Children's Fund (UNICEF) Multiple Indicator Cluster Surveys and OECD (2017).
- Mean years of schooling for adults ages 25 and older: UNESCO Institute for Statistics (2018), Barro and Lee (2016), ICF Macro Demographic and Health Surveys, UNICEF's Multiple Indicator Cluster Surveys and OECD (2017).
- Estimated earned income: Human Development Report Office estimates based on female and male shares of the economically active population, the ratio of the female to male wage in all sectors and gross national income in 2011 purchasing power parity (PPP) terms, and female and male shares of population from ILO (2018), UNDESA (2017), World Bank (2018) and IMF (2018).

Steps to calculate the Gender Development Index

There are four steps to calculating the GDI.

Step 1. Estimating the female and male earned incomes

To calculate estimated earned incomes, the share of the wage bill is calculated for each gender. The female share of the wage bill (S_f) is calculated as follows:

$$S_f = \frac{W_f/W_m \cdot EA_f}{W_f/W_m \cdot EA_f + EA_m}$$

where W_f/W_m is the ratio of female to male wage, EA_f is the female share of the economically active population and EA_m is the male share.

The male share of the wage bill is calculated as:

$$S_m = 1 - S_f$$

Estimated female earned income per capita ($GNIpc_f$) is obtained from GNI per capita ($GNIpc$), first by multiplying it by the female share of the wage bill, S_f, and then rescaling it by the female share of the population, $P_f = N_f/N$:

$$GNIpc_f = GNIpc \cdot S_f/P_f.$$

Estimated male earned income per capita is obtained in the same way:

$$GNIpc_m = GNIpc \cdot S_m/P_m$$

where $P_m = 1 - P_f$ is the male share of population.

Step 2. Normalizing the indicators

To construct the female and male HDI values, first the indicators, which are in different units, are transformed into indices and then dimension indices for each sex are aggregated by taking the geometric mean.

The indicators are transformed into indices on a scale of 0 to 1 using the same goalposts that are used for the HDI, except life expectancy at birth, which is adjusted for the average five-year biological advantage that women have over men.

Goalposts for the Gender Development Index in this Update

Indicator	Minimum	Maximum
Life expectancy at birth (years)		
Female	22.5	87.5
Male	17.5	82.5
Expected years of schooling (years)	0	18
Mean years of schooling (years)	0	15
Estimated earned income (2011 PPP $)	100	75,000

Note: For rationale on choice of minimum and maximum values, see *Technical note 1.*

Having defined the minimum and maximum values, the subindices are calculated as follows:

$$\text{Dimension index} = \frac{\text{actual value} - \text{minimum value}}{\text{maximum value} - \text{minimum value}}.$$

For education the dimension index is first obtained for each of the two subcomponents, and then the unweighted arithmetic mean of the two resulting indices is taken.

Step 3. Calculating the female and male Human Development Index values

The female and male HDI values are the geometric means of the three dimensional indices for each gender:

$$HDI_f = (I_{Health_f} \cdot I_{Education_f} \cdot I_{Income_f})^{1/3}$$

$$HDI_m = (I_{Health_m} \cdot I_{Education_m} \cdot I_{Income_m})^{1/3}$$

Step 4. Calculating the Gender Development Index

The GDI is simply the ratio of female HDI to male HDI:

$$GDI = \frac{HDI_f}{HDI_m}.$$

Example: Japan

Indicator	Female value	Male value
Life expectancy at birth (years)	87.1	80.7
Expected years of schooling (years)	15.17	15.29
Mean years of schooling (years)	12.87	12.53
Wage ratio (female/male)	0.7297	
Gross national income per capita (2011 PPP $)	38,986.15	
Share of economically active population	0.4322	0.5678
Share of population	0.51166	0.48834

Female wage bill:
$S_f = (0.7297 \cdot 0.4322) / [(0.7297 \cdot 0.4322) + 0.5678] = 0.35709$

Estimated female earned income per capita:
$GNIpc_f = 38{,}986.15 \cdot 0.35709 / 0.51166 = 27{,}208.6$

Male wage bill:
$S_m = 1 - 0.35709 = 0.64291$

Estimated male earned income per capita:
$GNIpc_m = 38{,}986.15 \cdot 0.64291 / 0.48834 = 51{,}326.1$

Female health index = (87.1 – 22.5) / (87.5 – 22.5) = 0.9938

Male health index = (80.7 – 17.5) / (82.5 – 17.5) = 0.9723

Female education index = [(15.17 / 18) + (12.87 / 15)] / 2 = 0.8504

Male education index = [(15.29 / 18) + (12.53 / 15)] / 2 = 0.8424

Estimated female earned income index:
[ln(27,208.6) – ln(100)] / [ln(75,000) – ln(100)] = 0.8468

Estimated male earned income index:
[ln(51,326.1) – ln(100)] / [(ln(75,000) – ln(100)] = 0.9427

Female HDI = $(0.9938 \cdot 0.8504 \cdot 0.8468)^{1/3} = 0.894$

Male HDI = $(0.9723 \cdot 0.8424 \cdot 0.9427)^{1/3} = 0.917$

GDI = 0.894 / 0.917 = 0.975

Note: Values are rounded.

GDI groups

The GDI groups are based on the absolute deviation of GDI from gender parity, $100 \cdot |GDI - 1|$. Countries with absolute deviation from gender parity of 2.5 percent or less are considered countries with high equality in HDI achievements between women and men and are classified as group 1. Countries with absolute deviation from gender parity of 2.5–5 percent are considered countries with medium-high equality in HDI achievements between women and men and are classified as group 2. Countries with absolute deviation from gender parity of 5–7.5 percent are considered countries with medium equality in HDI achievements between women and men and are classified as group 3. Countries with absolute deviation from gender parity of 7.5–10 percent are considered countries with medium-low equality in HDI achievements between women and men and are classified as group 4. Countries with absolute deviation from gender parity of more than 10 percent are considered countries with low equality in HDI achievements between women and men and are classified as group 5.

Technical note 4. Gender Inequality Index

The Gender Inequality Index (GII) reflects gender-based disadvantage in three dimensions—reproductive health, empowerment and the labour market—for as many countries as data of reasonable quality allow. It shows the loss in potential human development due to inequality between female and male achievements in these dimensions. It ranges from 0, where women and men fare equally, to 1, where one gender fares as poorly as possible in all measured dimensions.

The GII is computed using the association-sensitive inequality measure suggested by Seth (2009), which implies that the index is based on the general mean of general means of different orders—the first aggregation is by a geometric mean across dimensions; these means, calculated separately for women and men, are then aggregated using a harmonic mean across genders.

Data sources

- Maternal mortality ratio (*MMR*): UN Maternal Mortality Estimation Group (2017).
- Adolescent birth rate (*ABR*): UNDESA (2017).
- Share of parliamentary seats held by each sex (*PR*): IPU (2018).
- Population with at least some secondary education (*SE*): UNESCO Institute for Statistics (2018) and Barro and Lee (2016).
- Labour force participation rate (*LFPR*): ILO (2018).

Steps to calculate the Gender Inequality Index

There are five steps to calculating the GII.

Step 1. Treating zeros and extreme values

Because a geometric mean cannot be computed from zero values, a minimum value of 0.1 percent is set for all component indicators. Further, as higher maternal mortality suggests poorer maternal health, for the maternal mortality ratio the maximum value is truncated at 1,000 deaths per 100,000 births and the minimum value at 10. The rationale is that countries where maternal mortality ratios exceed 1,000 do not differ in their inability to create conditions and support for maternal health and that countries with 10 or fewer deaths per 100,000 births are performing at essentially the same level and that small differences are random. Sensitivity analysis of the GII is given in Gaye et al. (2010).

Step 2. Aggregating across dimensions within each gender group, using geometric means

Aggregating across dimensions for each gender group by the geometric mean makes the GII association sensitive (see Seth 2009).

For women and girls, the aggregation formula is:

$$G_F = \sqrt[3]{\left(\frac{10}{MMR}\cdot\frac{1}{ABR}\right)^{\frac{1}{2}}\cdot(PR_F\cdot SE_F)^{\frac{1}{2}}\cdot LFPR_F}\,, \qquad (1)$$

and for men and boys the formula is

$$G_M = \sqrt[3]{1\cdot(PR_M\cdot SE_M)^{\frac{1}{2}}\cdot LFPR_M}\,.$$

The rescaling by 0.1 of the maternal mortality ratio in equation 1 is needed to account for the truncation of the maternal mortality ratio at 10.

Step 3. Aggregating across gender groups, using a harmonic mean

The female and male indices are aggregated by the harmonic mean to create the equally distributed gender index

$$HARM(G_F, G_M) = \left[\frac{(G_F)^{-1} + (G_M)^{-1}}{2}\right]^{-1}.$$

Using the harmonic mean of within-group geometric means captures the inequality between women and men and adjusts for association between dimensions—that is, it accounts for the overlapping inequalities in dimensions.

Step 4. Calculating the geometric mean of the arithmetic means for each indicator

The reference standard for computing inequality is obtained by aggregating female and male indices using equal weights (thus treating the genders equally) and then aggregating the indices across dimensions:

$$G_{\overline{F,M}} = \sqrt[3]{\overline{Health}\cdot\overline{Empowerment}\cdot\overline{LFPR}}$$

$$\text{where } \overline{Health} = \left(\sqrt{\frac{10}{MMR}\cdot\frac{1}{ABR}} + 1\right)/2,$$

$$\overline{Empowerment} = \left(\sqrt{PR_F\cdot SE_F} + \sqrt{PR_M\cdot SE_M}\right)/2 \text{ and}$$

$$\overline{LFPR} = \frac{LFPR_F + LFPR_M}{2}.$$

$\overline{Health}$ should not be interpreted as an average of corresponding female and male indices but rather as half the distance from the norms established for the reproductive health indicators—fewer maternal deaths and fewer adolescent pregnancies.

Step 5. Calculating the Gender Inequality Index

Comparing the equally distributed gender index to the reference standard yields the GII,

$$1 - \frac{HARM(G_F, G_M)}{G_{\overline{F,M}}}.$$

Example: Sri Lanka

	Health		Empowerment		Labour market
	Maternal mortality ratio (deaths per 100,000 live births)	Adolescent birth rate (births per 1,000 women ages 15–19)	Share of seats in parliament (% held by women)	Population with at least some secondary education (%)	Labour force participation rate (%)
Female	30	14.1	5.8	82.6	35.1
Male	na	na	94.2	83.1	74.1
$\frac{F+M}{2}$	$\frac{\sqrt{\left(\frac{10}{30}\right)\cdot\left(\frac{1}{14.1}\right)}+1}{2}=0.5769$		$\frac{\sqrt{0.058\cdot 0.826}+\sqrt{0.942\cdot 0.831}}{2}=0.5518$		$\frac{0.351+0.741}{2}=0.546$

na is not applicable.

Using the above formulas, it is straightforward to obtain:

$$G_F: \sqrt[3]{\sqrt{\frac{10}{30}\cdot\frac{1}{14.1}}\cdot\sqrt{0.058\cdot 0.826}\cdot 0.351}=0.2277$$

$$G_M: \sqrt[3]{1\cdot\sqrt{0.942\cdot 0.831}\cdot 0.741}=0.8687$$

$$HARM(G_F, G_M): \left[\frac{1}{2}\left(\frac{1}{0.2277}+\frac{1}{0.8687}\right)\right]^{-1}=0.3608$$

$$G_{\overline{F,M}}: \sqrt[3]{0.5769\cdot 0.5518\cdot 0.546}=0.5581$$

GII: 1 – (0.3608/0.5581) = 0.354.

Technical note 5. Multidimensional Poverty Index

The global Multidimensional Poverty Index (MPI) identifies multiple deprivations at the household level in health, education and standard of living. It uses micro data from household surveys, and—unlike the Inequality-adjusted Human Development Index—all the indicators needed to construct the measure must come from the same survey. More details about the general methodology can be found in Alkire and Jahan (2018). Programmes (Stata do-files) for computing the MPI and its components for all countries with appropriate data will be available in due course at http://hdr.undp.org/en/content/mpi-statistical-programmes.

Data sources

- ICF Macro Demographic and Health Surveys.
- United Nations Children's Fund Multiple Indicator Cluster Surveys.
- For several countries, national household surveys with the same or similar content and questionnaires: Brazil, 2015 Pesquisa Nacional por Amostra de Domicílios; China, 2014 China Family Panel Studies; Ecuador, 2013–2014 Encuesta de Condiciones de Vida; Jamaica, 2014 Jamaica Survey of Living Conditions; Libya, 2014 Pan Arab Population and Family Health Survey; Mexico, 2016 Encuesta Nacional de Salud y Nutricion; Morocco, 2011 Pan Arab Population and Family Health Survey; South Africa, 2014–2015 National Income Dynamics Study; and Syrian Arab Republic, 2009 Pan Arab Population and Family Health Survey.

Methodology

The 2018 global MPI has the same functional form as in previous years, but some indicators have changed. It continues to use 10 indicators in three dimensions—health, education and standard of living—following the same dimensions and weights as the Human Development Index.

Each person is assigned a deprivation score according to his or her household's deprivations in each of the 10 indicators. The maximum deprivation score is 100 percent, with each dimension equally weighted; thus the maximum deprivation score in each dimension is 33.3 percent or more accurately 1/3. The health and education dimensions have two indicators each, so each indicator is weighted as 1/6. The standard of living dimension has six indicators, so each indicator is weighted as 1/18.

Dimension	Indicator	Deprived if...	Weight
Health	Nutrition	Any adult under age 70 or any child for whom nutritional information is available is undernourished. Adults over age 20 are considered undernourished if their body mass index is below 18.5 m/kg^2, individuals ages 15–19 are considered undernourished based on World Health Organization age-specific body mass index cutoffs and children are considered undernourished if the *z*-score of their height-for-age (stunting) or weight-for-age (underweight) is more than two standard deviations below the median of the reference population.	1/6
	Child mortality	Any child in the household has died in the five years preceding the survey. When a survey lacks information about the date of child deaths, deaths that occurred at any time are taken into account.[a]	1/6
Education	Years of schooling	No household member age 10 or older has completed six years of schooling.	1/6
	School attendance	Any school-age child[b] is not attending school up to the age at which he or she would complete class 8.	1/6
Standard of living	Electricity	The household has no electricity.	1/18
	Sanitation	The household does not have access to improved sanitation (according to Sustainable Development Goal guidelines), or it is improved but shared with other households. A household is considered to have access to improved sanitation if it has some type of flush toilet or latrine or ventilated improved pit or composting toilet that is not shared. When a survey uses a different definition of adequate sanitation, the survey report is followed.	1/18
	Drinking water	The household does not have access to an improved source of drinking water (according to Sustainable Development Goal guidelines), or safe drinking water is at least a 30-minute walk from home, roundtrip. A household is considered to have access to an improved source of drinking water if the source is piped water, a public tap, a borehole or pump, a protected well, a protected spring or rainwater. When a survey uses a different definition of safe drinking water, the survey report is followed.	1/18
	Housing	At least one of the household's three dwelling elements—floor, walls or roof—is made of inadequate materials—that is, the floor is made of natural materials and/or the walls and/or the roof are made of natural or rudimentary materials. The floor is made of natural materials such as mud, clay, earth, sand or dung; the dwelling has no roof or walls; the roof or walls are constructed using natural materials such as cane, palm, trunks, sod, mud, dirt, grass, reeds, thatch, bamboo or sticks or rudimentary materials such as carton, plastic or polythene sheeting, bamboo or stone with mud, loosely packed stones, uncovered adobe, raw or reused wood, plywood, cardboard, unburnt brick, or canvas or tent.	1/18
	Cooking fuel	The household cooks with dung, wood, charcoal or coal.	1/18
	Assets	The household does not own a car or truck and does not own more than one of the following assets: radio, television, telephone, computer, animal cart, bicycle, motorbike or refrigerator.	1/18

a. Information about child deaths is typically reported by women ages 15–49. When information from an eligible woman was not available, information from a man was used when the man reported no death in the household, and information was coded as missing when the man reported a death (because the date of the death was unknown).

b. Official school entrance age is from UIS.Stat (http://data.uis.unesco.org).

To identify multidimensionally poor people, the deprivation scores for each indicator are summed to obtain the household deprivation score. A cutoff of 1/3 is used to distinguish between poor and nonpoor people. If the deprivation score is 1/3 or higher, that household (and everyone in it) is considered multidimensionally poor. People with a deprivation score of 1/5 or higher but less than 1/3 are considered to be vulnerable to multidimensional poverty. People with a deprivation score of 1/2 or higher are considered to be in severe multidimensional poverty.

The headcount ratio, H, is the proportion of multidimensionally poor people in the population:

$$H = \frac{q}{n}$$

where q is the number of people who are multidimensionally poor and n is the total population.

The intensity of poverty, A, reflects the average proportion of the weighted component indicators in which multidimensionally poor people are deprived. For multidimensionally poor people only (those with a deprivation score c greater than or equal to 33.3 percent), the deprivation scores are summed and divided by the total number of multidimensionally poor people:

$$A = \frac{\sum_1^q c_i}{q}$$

where c_i is the deprivation score that the ith multidimensionally poor person experiences.

The deprivation score c_i of the ith multidimensionally poor person can be expressed as the sum of the weights associated with each indicator j (j = 1, 2, ..., 10) in which person i is deprived, $c_i = c_{i1} + c_{i2} + ... + c_{i10}$.

The MPI value is the product of two measures: the multidimensional poverty headcount ratio and the intensity of poverty:

$$MPI = H \cdot A$$

The contribution of dimension d to multidimensional poverty can be expressed as

$$Contrib_d = \frac{\sum_{j \in d} \sum_1^q c_{ij}}{n} / MPI$$

where d is health, education or standard of living.

Example using hypothetical data

Indicator	Indicator weight	Household 1	2	3	4
Household size		4	7	5	4
Health					
At least one member is undernourished	(⅓) ÷ 2 = 16.7%	0	0	1	0
One or more children have died	(⅓) ÷ 2 = 16.7%	1	1	0	1
Education					
No one has completed six years of schooling	(⅓) ÷ 2 = 16.7%	0	1	0	1
At least one school-age child not enrolled in school	(⅓) ÷ 2 = 16.7%	0	1	0	0
Living conditions					
No electricity	(⅓) ÷ 6 = 5.6%	0	1	1	1
No access to improved sanitation	(⅓) ÷ 6 = 5.6%	0	0	1	0
No access to an improved source of drinking water	(⅓) ÷ 6 = 5.6%	0	1	1	0
House built with inadequate materials	(⅓) ÷ 6 = 5.6%	0	0	0	0
Household cooks with dung, wood, charcoal or coal	(⅓) ÷ 6 = 5.6%	1	1	1	1
Household does not own a car or truck and does not own more than one of the following assets: radio, television, telephone, computer, animal cart, bicycle, motorbike or refrigerator.	(⅓) ÷ 6 = 5.6%	0	1	0	1
Results					
Individual deprivation score, *c* (sum of each deprivation multiplied by its weight) for each household member		22.2%	72.2%	38.9%	50.0%
Is the household multidimensionally poor ($c \geq 1/3$)?		No	Yes	Yes	Yes

Note: 1 indicates deprivation in the indicator; 0 indicates nondeprivation.

Weighted deprivations:

- Household 1: $(1 \cdot 16.67) + (1 \cdot 5.56) = 22.2$ percent.
- Household 2: 72.2 percent.
- Household 3: 38.9 percent.
- Household 4: 50.0 percent.

Based on this hypothetical population of four households:

Headcount ratio *(H)* =

$$\left(\frac{0+7+5+4}{4+7+5+4}\right) = 0.80$$

(80 percent of people are multidimensionally poor).

Intensity of poverty *(A)* =

$$\frac{(72.2 \cdot 7) + (38.9 \cdot 5) + (50.0 \cdot 4)}{(7+5+4)} = 56.3 \text{ percent}$$

(the average multidimensionally poor person is deprived in 56.3 percent of the weighted indicators).

$$\textbf{MPI} = H \cdot A = 0.8 \cdot 0.563 = 0.450.$$

Contribution of deprivations in:

Health:

$$contrib_1 = \frac{16.67 \cdot 5 + 16.67 \cdot (7+4)}{4+7+5+4} / 0.450 = 29.6 \text{ percent}$$

Education:

$$contrib_2 = \frac{16.67 \cdot (7+4) + 16.67 \cdot 7}{4+7+5+4} / 0.450 = 33.3 \text{ percent}$$

Living conditions:

$$contrib_3 = \frac{5.56 \cdot (7 \cdot 4 + 5 \cdot 4 + 4 \cdot 3)}{} / 0.450 = 37.1 \text{ percent.}$$

Calculating the contribution of each dimension to multidimensional poverty provides information that can be useful for revealing a country's deprivation structure and can help with policy targeting.

Technical note 6. Human development dashboards 1–5

This Update includes colour-coded dashboards on five topics: quality of human development, life-course gender gap, women's empowerment, environmental sustainability and socioeconomic sustainability.

The dashboards allow partial grouping of countries by indicator—rather than complete grouping by a composite measure, such as the Human Development Index (HDI)—that combines multiple indicators after making them commensurable. A complete grouping depends on how component indicators are combined, but a partial grouping does not require assumptions about normalization, weighting or the functional form of the composite index. A partial grouping may depend on the predefined values used as thresholds for grouping, such as what is considered good performance or a target to be achieved.

For each indicator in the dashboards, countries are divided into three groups of approximately equal size (terciles): the top third, the middle third and the bottom third.[2] The intention is not to suggest thresholds or target values for the indicators but to allow a crude assessment of a country's performance relative to others. A country that is in the top third performs better than at least two-thirds of countries, a country that is in the middle third performs better than at least one-third of countries but worse than at least one-third, and a country that is in the bottom third performs worse than at least two-thirds of countries. For

indicators expressed as female to male ratio, countries with a value near 1 are classified as top performers, and deviations from parity are treated equally regardless of which gender is overachieving.

Three-colour coding is used to visualize the partial grouping of countries by indicator—a simple tool to help users immediately discern a country's performance. The colour-coding scale graduates from darkest for the top third to medium for the middle third to lightest for the bottom third.

Aggregates for human development categories, regions, least developed countries, small island developing states, Organisation for Economic Co-operation and Development countries and the world are coloured based on which grouping their values fall into for each indicator.

Dashboard 1. Quality of human development

Dashboard 1 contains 13 indicators associated with the quality of health, education and standard of living. The three indicators on quality of health are lost health expectancy, number of physicians and number of hospital beds. The six indicators on quality of education are pupil–teacher ratio in primary schools; primary school teachers trained to teach; proportion of schools with access to the Internet; and Programme for International Student Assessment (PISA) scores in mathematics, reading and science. The four indicators on quality of standard of living are proportion of employment that is in vulnerable employment, proportion of rural population with access to electricity, proportion of population using improved drinking-water sources and proportion of population using improved sanitation facilities.

Aggregates are not presented for proportion of schools with access to the Internet and PISA scores.

The following table shows the ranges of values that define tercile groups and the number of countries in each tercile group for each indicator in dashboard 1.

Observed ranges of values and number of countries in each tercile group, by indicator, dashboard 1: quality of human development

	Top third		Middle third		Bottom third		
Indicator	Range	Number of countries	Range	Number of countries	Range	Number of countries	Countries with missing values
Lost health expectancy (%)	≤11.5	66	11.5–12.0	49	>12.0	68	12
Physicians (per 10,000 people)	≥25.0	58	5.5–25.0	60	<5.5	57	20
Hospital beds (per 10,000 people)	≥35	63	15–35	67	<15	58	7
Pupil-teacher ratio, primary school (pupils per teacher)	≤15	59	15–25	53	>25	55	27
Primary school teachers trained to teach (%)	≥95	47	75–95	31	<75	39	78
Proportion of schools with access to the Internet (%)	≥90	38	50–90	17	<50	29	111
Programme for International Student Assessment (PISA) score, mathematics	≥495	20	425–495	25	<425	22	128
Programme for International Student Assessment (PISA) score, reading	≥495	24	435–495	20	<435	23	128
Programme for International Student Assessment (PISA) score, science	≥495	25	435–495	21	<435	21	128
Vulnerable employment (% of total employment)	≥20.0	60	20.0–45.0	57	>45.0	63	15
Rural population with access to electricity (%)	=100.0	98	75.0-100.0	32	<75.0	61	4
Population using improved drinking-water sources (%)	≥98.0	67	85.0–98.0	65	<85.0	61	2
Population using improved sanitation facilities (%)	≥95.0	70	65.0–95.0	62	<65.0	61	2

Dashboard 2. Life-course gender gap

Dashboard 2 contains 12 indicators that display gender gaps in choices and opportunities over the life course—childhood and youth, adulthood and older age. The five indicators on childhood and youth are sex ratio at birth; gross enrolment ratios in pre-primary, primary and secondary school; and youth unemployment rate. The six indicators on adulthood are population with at least some secondary education, total unemployment rate, female share of employment in nonagriculture, share of seats in parliament held by women, and time spent on unpaid domestic chores and care work (expressed two ways). The indicator on older age is old-age pension recipients. Eight indicators are presented as a ratio of female to male values, and three are presented as values for women only. Sex ratio at birth (male to female births) is an exception to grouping by tercile—countries are divided into two groups: the natural group (countries with a value of 1.04–1.07, inclusive) and the gender-biased group (all other countries).

Deviations from the natural sex ratio at birth have implications for population replacement levels, suggest possible future social and economic problems and may indicate gender bias.

Aggregates are not presented for time spent on unpaid domestic chores and care work.

The following table shows the ranges of values that define each tercile group and the number of countries in each tercile group for each indicator in dashboard 2.

Observed ranges of values and number of countries in each tercile group, by indicator, dashboard 2: life-cycle gender gap

	Top third		Middle third		Bottom third		
Indicator	Range	Number of countries	Range	Number of countries	Range	Number of countries	Countries with missing values
Sex ratio at birth (male to female births)	1.04–1.07	135	—	—	<1.04, >1.07	50	10
Gross enrolment ratio, pre-primary (female to male ratio)	0.99–1.01	58	0.97–0.99, 1.01–1.03	50	<0.97, >1.03	57	30
Gross enrolment ratio, primary (female to male ratio)	0.99–1.01	81	0.97–0.99, 1.01–1.03	45	<0.97, >1.03	52	17
Gross enrolment ratio, secondary (female to male ratio)	0.98–1.02	56	0.92–0.98, 1.02–1.08	60	<0.92, >1.08	50	29
Youth unemployment rate (female to male ratio)	0.95–1.05	50	0.85–0.95, 1.05–1.15	65	<0.85, >1.15	65	15
Population with at least some secondary education (female to male ratio)	0.95–1.05	69	0.80–0.95, 1.05–1.20	53	<0.80, >1.20	43	30
Total unemployment rate (female to male ratio)	0.90–1.10	35	0.75–0.90, 1.10–1.25	58	<0.75, >1.25	87	15
Share of employment in nonagriculture, female (% of total employment in nonagriculture)	≥45.0	75	40.0–45.0	40	<40.0	65	15
Share of seats in parliament (% held by women)	≥25.0	68	15.0–25.0	67	<15.0	58	2
Time spent on unpaid domestic chores and care work, women ages 15 and older (% of 24-hour day)	≤15.0	17	15.0–20.0	41	>20.0	15	122
Time spent on unpaid domestic chores and care work (female to male ratio)	≤2.0	23	2.0–3.0	25	>3.0	25	122
Old-age pension recipients (female to male ratio)	0.99–1.00	35	0.80–0.99, 1.00–1.20	8	<0.80, >1.20	20	132

Dashboard 3. Women's empowerment

Dashboard 3 contains 13 woman-specific empowerment indicators that allow empowerment to be compared across three dimensions: reproductive health and family planning, violence against girls and women, and socioeconomic empowerment. The six indicators on reproductive health and family planning are coverage of at least one antenatal care visit, proportion of births attended by skilled health personnel, maternal mortality ratio, adolescent birth rate, contraceptive prevalence (any method) and unmet need for family planning. The three indicators on violence against girls and women are women married by age 18, violence against women ever experienced from an intimate partner and violence against women ever experienced from a nonintimate partner. The four indicators on socioeconomic empowerment are female share of graduates in science, mathematics, engineering, manufacturing and construction at tertiary level; female share of employment in senior and middle management; women with account at financial institution or with mobile money-service provider; and mandatory paid maternity leave.

Most countries have at least one indicator in each tercile, which implies that women's empowerment is unequal across indicators and across countries.

Aggregates are not presented for female share of employment in senior and middle management.

The following table shows the ranges of values that define each tercile group and the number of countries in each tercile group for each indicator in dashboard 3.

Observed ranges of values and number of countries in each tercile group, by indicator, dashboard 3: women's empowerment

Indicator	Top third		Middle third		Bottom third		Countries with missing values
	Range	Number of countries	Range	Number of countries	Range	Number of countries	
Antenatal care coverage, at least one visit (%)	≥97.5	47	92.0–97.5	46	<92.0	52	50
Proportion of births attended by skilled health personnel (%)	≥99.0	64	90.0–99.0	39	<90.0	60	32
Maternal mortality ratio (deaths per 100,000 live births)	≤25	64	25–150	59	>150	59	13
Adolescent birth rate (births per 1,000 women ages 15–19)	≤20.0	62	20.0–60.0	61	>60.0	62	10
Contraceptive prevalence, any method (% of married or in-union women of reproductive age, 15–49 years)	≥60.0	59	40.0–60.0	39	<40.0	54	43
Unmet need for family planning (% of married or in-union women of reproductive age, 15–49 years)	≤12.5	43	12.5–23.0	41	>23.0	42	69
Women married by age 18 (% of women ages 20–24 who are married or in union)	≤15	42	15–27	43	>27	40	70
Violence against women ever experienced, intimate partner (% of female population ages 15 and older)	≤20.0	30	20.0–30.0	41	>30.0	36	88
Violence against women ever experienced, nonintimate partner (% of female population ages 15 and older)	≤4.0	23	4.0–8.0	16	>8.0	17	139
Female share of graduates in science, mathematics, engineering, manufacturing and construction at tertiary level (%)	≥15.0	38	10.0–15.0	38	<10.0	30	89
Female share of employment in senior and middle management (%)	≥35.0	27	30.0–35.0	21	<30.0	33	114
Women with account at financial institution or with mobile money-service provider (% of female population ages 15 and older)	≥75.0	50	40.0–75.0	48	<40.0	58	39
Mandatory paid maternity leave (days)	≥105	62	90–105	57	<90	59	17

Dashboard 4. Environmental sustainability

Dashboard 4 contains 10 indicators that cover environmental sustainability and environmental threats. The seven indicators on environmental sustainability are fossil fuel energy consumption, renewable energy consumption, carbon dioxide emissions (expressed two ways), forest area (expressed two ways) and fresh water withdrawals. The three indicators on environmental threats are mortality rate attributed to household and ambient air pollution and to unsafe water, sanitation and hygiene services and the International Union for Conservation of Nature's Red List Index, which measures aggregate extinction risk across groups of species.

The percentage of total land area under forest is intentionally left without colour because it is meant to provide context for the indicator on change in forest area. Aggregates are not presented for the Red List Index indicator.

The following table shows the ranges of values that define each tercile group and the number of countries in each tercile group for each indicator in dashboard 4.

Observed ranges of values and number of countries in each tercile group, by indicator, dashboard 4: environmental sustainability

Indicator	Top third		Middle third		Bottom third		Countries with missing values
	Range	Number of countries	Range	Number of countries	Range	Number of countries	
Fossil fuel energy consumption (% of total energy consumption)	≤60.0	44	60.0 to 85.0	46	>85.0	48	57
Renewable energy consumption (% of total final energy consumption)	≥40.0	63	15.0 to 40.0	56	<15.0	74	2
Carbon dioxide emissions, per capita (tonnes)	≤1.0	62	1.0 to 4.5	67	>4.5	64	2
Carbon dioxide emissions (kg per 2011 PPP $ of GDP)	≤0.15	51	0.15 to 0.25	74	>0.25	62	8
Forest area (% of total land area)	—	—	—	—	—	—	—
Forest area, change (%)	≥4.0	62	–3.0 to 4.0	56	<–3.0	68	9
Fresh water withdrawals (% of total renewable water sources)	≤3.0	36	3.0 to 14.0	34	>14.0	36	89
Mortality rate attributed to household and ambient air pollution (per 100,000 population)	≤45.0	59	45.0 to 115.0	63	>115.0	61	12
Mortality rate attributed to unsafe water, sanitation and hygiene services (per 100,000 population)	≤0.5	68	0.5 to 6.0	52	>6.0	63	12
Red List Index (value)	≥0.925	56	0.825 to 0.925	73	<0.825	66	0

Dashboard 5. Socioeconomic sustainability

Dashboard 5 contains 11 indicators that cover economic and social sustainability. The six indicators on economic sustainability are adjusted net savings, total debt service, gross capital formation, skilled labour force, export concentration index, and research and development expenditure. The five indicators on social sustainability are military expenditure, ratio of education and health expenditure to military expenditure, change in overall loss in HDI value due to inequality, change in Gender Inequality Index value and change in income quintile ratio.

The military expenditure indicator is intentionally left without colour because it is meant to provide context for the indicator on education and health expenditure. Aggregates are not presented for export concentration index and change in income quintile ratio.

The following table shows the ranges of values that define each tercile group and the number of countries in each tercile group for each indicator in dashboard 5.

Observed ranges of values and number of countries in each tercile group, by indicator, dashboard 5: socioeconomic sustainability

Indicator	Top third		Middle third		Bottom third		Countries with missing values
	Range	Number of countries	Range	Number of countries	Range	Number of countries	
Adjusted net savings (% of GNI)	≥12.0	57	3.0 to 12.0	45	<3.0	51	42
Total debt service (% of exports of goods, services and primary income)	≤6.0	38	6.0 to 16.0	38	>16.0	41	78
Gross capital formation (% of GDP)	≥25.0	67	20.0 to 25.0	58	<20.0	49	21
Skilled labour force (% of labour force)	≥75.0	51	40.0 to 75.0	41	>40.0	49	54
Concentration index (exports) (value)	≤0.200	61	0.200 to 0.400	72	>0.400	58	4
Research and development expenditure (% of GDP)	≥1.0	39	0.3 to 1.0	51	<0.3	36	69
Military expenditure (% of GDP)	—	—	—	—	—	—	—
Ratio of education and health expenditure to military expenditure	≥10.0	40	6.0 to 10.0	39	<6.0	38	78
Overall loss in HDI value due to inequality, average annual change (%)	≤–2.0	51	–2.0 to –0.5	37	>–0.5	44	63
Gender Inequality Index, average annual change (%)	≤–2.0	47	–2.0 to –1.0	50	>–1.0	47	51
Income quintile ratio, average annual change (%)	≤–1.0	45	–1.0 to 0.0	38	>0.0	41	71

Notes

1. The inequality aversion parameter affects the degree to which lower achievements are emphasized and higher achievements are de-emphasized.

2. For some skewed distributions the groups differ in sizes. In addition, for the sex ratio at birth indicator, countries are divided into two groups (see section on dashboard 2).

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